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Analysis of Aircraft Agility on Maximum Performance Maneuvers

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This paper presents an analysis of aircraft agility as an inverse simulation problem. The analytical framework is based on the differential geometry of the trajectory and on the complete set of motion equations for a rigid airframe. A local optimization method is adopted and the solution is obtained in terms of the trajectory properties subjected to assigned constraints. Three applications are presented, namely a rapid deceleration maneuver, a maximum performance turn, and a reverse turn. The results show that the method is flexible enough to deal with the maximization of the agility components, for some assigned characteristics of the trajectory, and to determine the control actions for maneuvers that are usually proposed for evaluating the aircraft agility.

Nomenclature

$\boldsymbol{A}$	= agility vector
A_t, A_n, A_b	= agility components in F_F
$\boldsymbol{a}$	= acceleration vector
F_B	= body-fixed frame
F_F	= intrinsic frame
h	= altitude
k_1, k_2	= curvature and torsion
M_T	= test Mach number
P_S	= specific excess power
p, q, r	= angular velocity components
$\boldsymbol{R}$	= position vector of the aircraft c.g.
S	= cost function
s	= curvilinear abscissa
t	= time
$\boldsymbol{t}$, $\boldsymbol{n}$, $\boldsymbol{b}$	= tangential, normal, and binormal unit vectors
$\boldsymbol{u}$	= control vector
V	= velocity modulus
V_c	= corner velocity
$\boldsymbol{v}$	= velocity vector
$\boldsymbol{x}$	= state vector
$\boldsymbol{y}$	= output vector
α	= angle of attack
β	= sideslip angle
γ	= flight-path angle
δ_A	= aileron angle
δ_E	= elevator angle
δ_R	= rudder angle
δ_T	= throttle position
δ_{t_c}	= actual power level
ϕ, θ, ψ	= Euler angles
$\boldsymbol{\omega}$	= angular velocity vector

Introduction

RECENTLY, research activity has been carried out to evaluate the agility of high-performance aircraft and helicopters, whereas, even as these investigations are rapidly expanding, a commonly accepted definition of the appropriate metrics of this important flight-quality parameter is still a matter of debate. In Ref. 1 the proposed metrics to assess the agility of fighter aircraft are classified according to the type of motion, namely axial, longitudinal, and lateral metrics for translational, pitching, and rolling motion, respectively, and the time scale. To this aim the nonlinear simulation of an F-18 fighter model is used. As far as the time scale is considered and regardless of the motion variables involved, the simulation study shows that a convenient classification of the agility metrics is based on grouping them in two basic sets. The first set groups those metrics that can be addressed as functional agility, whereas the second set groups the transient agility metrics. Each group corresponds to a different time scale. In particular, the functional, long-time-scale agility quantifies how well the aircraft executes rapid rotations of the velocity vector. The time scale is about 20–30 s. Functional agility metrics are used in Ref. 2 to compare the turning performances of different fighter aircraft models. To reduce the sensitivities of the computed metrics to pilot input techniques and flight-control systems of different aircraft, optimal test trajectories are determined where a global performance index is minimized subject to path constraints.

The transient or inherent agility is, on the other hand, related to short-time rotational motions and transition between extreme specific power levels. In this case the time scale is about 1–3 s and the metrics of this second group are slightly influenced by aspects such as a pilot's skill and aircraft flying qualities.

A rational concept of transient agility, defined as a property that characterizes the time rate of change of the acceleration state, was developed by Mazza.[3] By considerations from the differential geometry, the components of the agility vector are expressed in the intrinsic or Frenet reference system, in terms of kinematic variables, i.e., arc length, curvature, and torsion

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and their time derivatives. The relationship between agility and dynamic parameters is discussed in detail in Ref. 4, where the agility components are formulated in terms of the forces and their derivatives, and are also related to kinematic variables. Because the agility depends on both the changes in force magnitude and the rotation of the force vector, the attitude dynamics of the aircraft is to be taken into consideration to determine the angular rate of the maneuver plane, i.e., the plane spanned by the velocity and acceleration vectors, and the control actions necessary to realize specified agility characteristics. In the same framework, Burazanis et al.[5] focused on the use of the Frenet system and the concept of a maneuver plane when short-term agility features are dealt with. In that study a point-mass dynamic model is introduced, which is extended by additional states explicitly related to the maneuver plane, and a flight optimization problem is formulated that emphasizes agility.

A measure of the inherent agility of different helicopter models by inverse simulation of standard trajectories is presented by Thomson.[6] We recall here that by inverse simulation, one means the determination of the control actions needed for assigned flight paths. An agility rating is awarded to a model on the basis of the values assumed by a quadratic performance function of state and control variables over all of the maneuvers relevant to the helicopter's role. The inverse method uses a differential approach, the starting point for which is the evaluation of the velocities and accelerations along the assigned flight path.

The present study is an analysis of the airframe agility as an inverse simulation problem that is solved by a local optimization technique.[7] Following the findings of Refs. 3 and 4, this approach uses concepts from the differential geometry to set an analytical framework for the derivation of agility parameters that are expressed in terms of trajectory-related variables. An advantage of using a local optimization method is the possibility of determining the control laws for a flight path where a component of the agility in the Frenet frame is maximized and suitable constraints can be enforced. The constraints, compatible with the aircraft model, may be imposed either on the other agility components or on the kinematic parameters of the flight path. Furthermore, because particular maneuvers have been suggested for evaluating some maximum performances of an aircraft, the proposed way for solving the corresponding inverse problem leads to the determination of the attained agility values.

We take advantage of the rigorous definition of the short-term agility as the time derivative of the acceleration vector.[4] This peculiar metric, based on the instantaneous characteristics of the c.g. motion, is best expressed in terms of the evolution of the natural coordinates of the c.g. along the trajectory and appears very suitable when dealing with the appreciation of the short-time maneuvering performances of an aircraft. Because the transient agility is basically related to defined configurational parameters of the vehicle and is independent of any pilot influence, the proposed approach is more focused on the analysis of design parameters in terms of maximum performances rather than on the selection of test maneuvers for actual flight or a piloted simulation.

Our purpose is to show how the short-term metrics and the use of a solution method of inverse problems can lead to the determination of the maximum attainable agility components of an airplane and, at the same time, to the calculation of the related time histories of the state variables and the corresponding control actions. In particular, once a vector of constrained outputs is given, the difference between the number n of the control variables and the m assigned outputs, being in any case $n \geq m$, is termed the *degree of redundancy*, whereas the situation $n = m$ is referred to as a *nominal problem.* A nonzero degree of redundancy allows for a suitable cost function to be introduced in such a way that either the variations of the agility vector components can follow a desired path or the maximum value of one of the components can be reached. Furthermore, the limitations on the control deflections can be implemented by enforcing inequality constraints.

As compared to agility research, where test trajectories are generated by global optimization,[2] the proposed technique provides a computationally fast solution, whereas both trajectory and attitude dynamics are incorporated in the aircraft model. Also, the inverse simulation algorithm appears to be more flexible in comparison to the differential method adopted in Ref. 6. In fact, the differential technique applies to nominal problems only, where the complete specification of the flight path as a function of time is required. More importantly, by local optimization the resulting motion can be given certain properties that are directly characterized in terms of flight-path-related parameters, as in the case of the agility components.

A brief presentation of the analytical and numerical method that will be applied is given and the model of the F-16 fighter aircraft used in the calculations is recalled. A few applications concerning the evaluation of tangential, normal, and torsional agility are then presented and analyzed.

Analysis

We begin this section by recalling the development of the agility metric defined earlier. Following the differential geometry approach[8] we refer to the Frenet or intrinsic frame F_F, which has unit vectors $\boldsymbol{t} = \mathrm{d}\boldsymbol{R}/\mathrm{d}s$ along the tangent to the trajectory, positive in the flight direction; $\boldsymbol{n} = (\mathrm{d}\boldsymbol{t}/\mathrm{d}s)/|\mathrm{d}\boldsymbol{t}/\mathrm{d}s|$ along the instantaneous radius of curvature, positive toward the center of curvature; and $\boldsymbol{b} = \boldsymbol{t} \wedge \boldsymbol{n}$ along the binormal. The agility vector, defined as the rate of change of the maneuver state, is $\boldsymbol{A} = \mathrm{d}\ddot{\boldsymbol{R}}/\mathrm{d}t$, where the acceleration $\ddot{\boldsymbol{R}}$ can be expressed in the familiar form $\ddot{\boldsymbol{R}} = \ddot{s}\boldsymbol{t} + \dot{s}^2 k_1 \boldsymbol{n}$, k_1 being the curvature of the trajectory. When the so-called *Frenet Formulae*[8] are used

$$\frac{\mathrm{d}\boldsymbol{t}}{\mathrm{d}t} = k_1\dot{s}\boldsymbol{n}, \qquad \frac{\mathrm{d}\boldsymbol{n}}{\mathrm{d}t} = -k_1\dot{s}\boldsymbol{t} + k_2\dot{s}\boldsymbol{b}, \qquad \frac{\mathrm{d}\boldsymbol{b}}{\mathrm{d}t} = -k_2\dot{s}\boldsymbol{n} \tag{1}$$

where k_2 is the torsion defined as the rate of change of the osculating plane about $\boldsymbol{t}$, the components of $\boldsymbol{A}$ in the intrinsic frame are written

$$\boldsymbol{A} = (A_t, A_n, A_b)^T = \begin{bmatrix} \ddot{V} - V^3k_1^2 \\ 3V\dot{V}k_1 + V^2\dot{k}_1 \\ V^3k_1k_2 \end{bmatrix} \tag{2}$$

where A_t, A_n, and A_b are, respectively, the axial, curvature, and torsion agility, and $V = \dot{s}$ is the velocity modulus. As an observation, the torsion is zero for a planar flight path and, for a constant value of k_1 and k_2, the flight trajectory is a cylindrical helix.

The six-degree-of-freedom model of the F-16 fighter aircraft reported in Ref. 9 includes the engine dynamics, where the thrust response is modeled as a first-order lag and the lag time constant is a function of the actual and commanded power levels. The nonlinear set of governing equations formulated in the body-fixed reference frame F_B for flat-Earth and zero ambient wind is standard and written in the concise form

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}), \qquad \boldsymbol{x}(t = 0) = \boldsymbol{x}_e \tag{3}$$

where the subscript e indicates steady-state conditions. The 11 elements of the state vector $\boldsymbol{x} = (\boldsymbol{v}, \boldsymbol{\omega}, \phi, \theta, \psi, h, \delta_{T_c})^T$ are the components of the linear and angular velocity vectors $\boldsymbol{v}$ and $\boldsymbol{\omega}$; the Euler angles ϕ, θ, and ψ; the altitude h in the inertial frame; and the actual power level δ_{T_c}. The control vector $\boldsymbol{u} \in R^4$ is $\boldsymbol{u} = (\delta_E, \delta_A, \delta_R, \delta_T)^T$, where the elements are elevator, aileron, and rudder angles, and throttle position, respectively.

The nonlinear aerodynamic model uses tabulated data of force and moment coefficients as functions of the aerodynamic angles α and β and of the control angles. The effects of the

F-16 leading-edge flap on the aerodynamic coefficients are incorporated in the data, whereas the flap actuator dynamics are neglected. The model applies in the subsonic range, for -10 deg $\le \alpha \le 45$ deg and $-30 \le \beta \le 30$ deg. The speed brake of the aircraft is modeled by considering additional terms, depending on α and expressed as in Ref. 10, for the force and moment coefficients.

Finally, for the output vector $\boldsymbol{y} \in R^m$, we write

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \tag{4}$$

When the agility components are determined as output variables by Eq. (2), we have $V = |\boldsymbol{v}|$, and the curvature and torsion are conveniently expressed as functions of the components in F_B of the velocity, the inertial acceleration $\boldsymbol{a}$, and its time derivative $\dot{\boldsymbol{a}}$ as follows[8]:

$$k_1 = (1/V^2)(|\boldsymbol{a}|^2 - \dot{V}^2)^{1/2} \tag{5}$$

$$k_2 = \frac{\det\{\boldsymbol{v}, \boldsymbol{a}, \dot{\boldsymbol{a}}\}}{V^2(|\boldsymbol{a}|^2 - \dot{V}^2)} \tag{6}$$

where, as usual, $\boldsymbol{a} = \dot{\boldsymbol{v}} + \tilde{\boldsymbol{\omega}}\boldsymbol{v}$ and $\dot{\boldsymbol{a}} = \ddot{\boldsymbol{v}} + \dot{\tilde{\boldsymbol{\omega}}}\boldsymbol{v} + 2\tilde{\boldsymbol{\omega}}\dot{\boldsymbol{v}} + \tilde{\boldsymbol{\omega}}\tilde{\boldsymbol{\omega}}\boldsymbol{v}$.

The inverse simulation problem is solved by a local optimization technique.[7] We assume $\boldsymbol{u}$ to be a step-constant function $\boldsymbol{u}(t) = \boldsymbol{u}_j$, $t_{j-1} < t \le t_j$, so that Eq. (3) can be numerically solved, once $\boldsymbol{x}_{j-1}$ is known, as follows:

$$\boldsymbol{x}_j = \boldsymbol{F}(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j) \tag{7}$$

Therefore, Eq. (4) is recast in the form

$$\boldsymbol{y}_j = \boldsymbol{g}[\boldsymbol{F}(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j), \boldsymbol{u}_j] = \boldsymbol{h}(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j) \tag{8}$$

The inverse problem is solved when the discretized input $\boldsymbol{u}_j^*$ is determined at each time step, for an assigned output $\boldsymbol{y}_j^D$, as the inverse of the implicit function

$$\boldsymbol{y}_j^D = \boldsymbol{h}(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j^*) \tag{9}$$

In the redundant case which, in this study, occurs when the number of given outputs is less than four, the constrained optimization problem represented by the minimization of the performance index $S(\boldsymbol{x}_{j-1}, \boldsymbol{u}_j)$ subjected to Eq. (9) is solved by a sequential quadratic programming (SQP) algorithm. S is a scalar, positive-semidefinite function. $S = 0$ when the problem is nominal ($m = 4$) and the resulting solution simply satisfies the constraints given by Eq. (9). Further details on the preceding method, including the SQP procedure that consists of solving a quadratic programming subproblem and of updating a Hessian matrix, are reported in Ref. 7.

Results

In the following text a number of situations were considered to demonstrate the practicality of adopting the inverse simulation technique in aircraft agility evaluation. The three components of $\boldsymbol{A}$, in order, are considered and calculated in subsonic flight conditions.

The tangential agility, at decreasing speed, is first dealt with according to three different approaches. Two of these approaches correspond to assuming that $A_{t_{max}}$ is calculated while keeping A_n and A_b both exactly zero, i.e., for curvature and torsion of the trajectory both vanishing. The third case corresponds to the evaluation of $A_{t_{max}}$ while $A_n \ne 0$.

In this respect, the preceding procedure was intended to analyze situations where the number of factors affecting the aircraft performances was kept to a minimum, even though keeping A_n and A_b constantly equal to zero would be difficult in an actual maneuver. However, from the simple form of Eq. (2) we observe that when axial agility maneuvers involving aircraft deceleration are taken into consideration, any curvature of the trajectory acts to further decrease the negative value of A_t. This effect may be relevant as will be seen.

A case is carried out that is thought to be particularly significant because it demonstrates how our results on A_t are related to those in Ref. 1, where flight simulations were performed and a different agility metric was considered. The flight is in the symmetry plane, i.e., k_2 is identically equal to zero, and h is constant. The speed is increased from an initial trimmed state by reaching the maximum throttle value, an assigned test speed is attained, the thrust is abruptly decreased to zero, and the speed brakes can be extended. This inverse problem is nominal because we assign $k_1 = 0$, and the only unknown is δ_E. For this case, Fig. 1 reports the time histories of some meaningful quantities, namely the agilities A_t and A_n, the states V and α, the control actions δ_E and δ_T, and the actual thrust level δ_{T_c}. Figures 2 and 3 show the values of $A_{t_{max}}$, which are obtained following the indicated flight procedure, as functions of the assigned test Mach number M_T, of the altitude, and of the brake action. Note in Fig. 1 the time lag and the

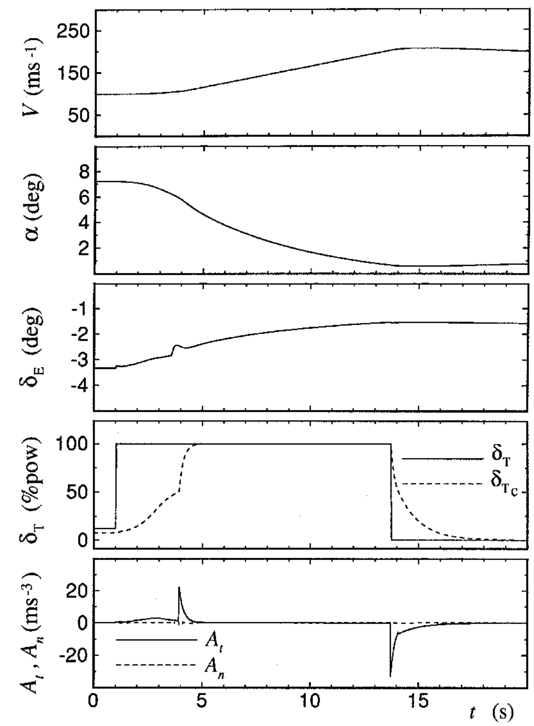

Fig. 1 Time histories for assigned $\delta_T(t)$, $k_1 = 0$. Speed brake not deflected, $M_T = 0.6$, $h = 0$.

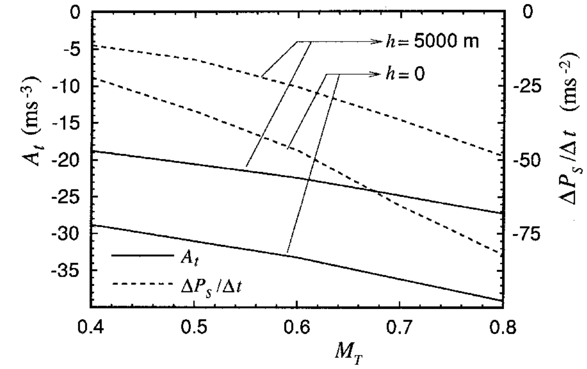

Fig. 2 $A_{t_{max}}$ and $\Delta P_S/\Delta t$ vs M_T at two altitudes; $k_1 = 0$. Speed brake not deflected.

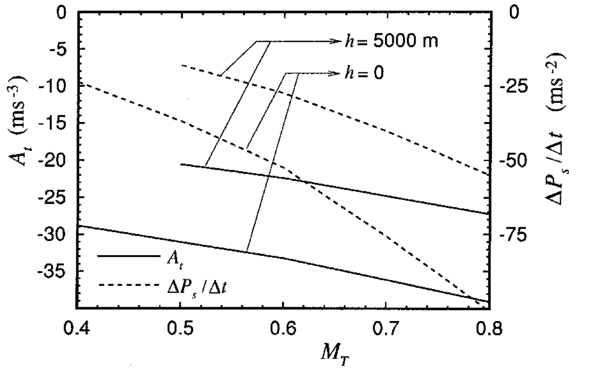

Fig. 3 $A_{t_{max}}$ and $\Delta P_S/\Delta t$ vs M_T at two altitudes; $k_1 = 0$. Speed brake deflected.

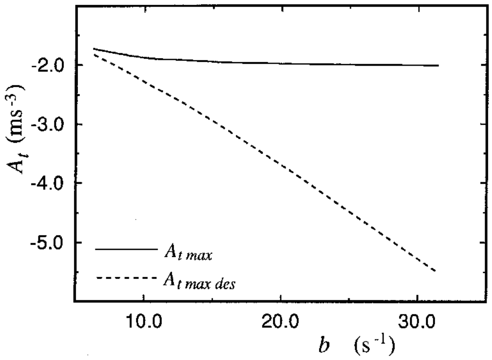

Fig. 4 $A_{t_{max}}$ and $A_{t_{max_{des}}}$ vs b; $M_e = 0.6$, $h = 0$.

effect of the afterburner ignition on A_t that takes place at t = 4 s.

At sea level and for h = 5000 m, the obtained tangential agility vs M_T is shown in Fig. 2, when the speed brake is retracted. According to Ref. 1, the power-loss parameter $\Delta P_S/\Delta t$ is defined as the increment of specific excess power in going from a maximum power/minimum drag condition to a minimum power/maximum drag condition divided by the time necessary to complete the transition. This parameter can be chosen as a different and practical agility metric. The time difference Δt is between the instant when the minimum deceleration occurs and the time when the value of M_T is obtained.

According to our computations at sea level and for M_T ranging from 0.4 to 0.8, Δt changes between 6.78 and 4.40 s, respectively. For h = 5000 m, the time increments Δt are of the order of 10% greater with respect to the values at h = 0 and at the same M_T. Figure 2 also shows the power-loss parameter and it can be immediately verified that the two sets of curves give comparable results only at higher M_T. This can be realized because $\Delta P_S/\Delta t$ evaluates the rate of change of the maneuver state over the entire time length to perform the prescribed task. On the other hand, the definition of A_t leads to the calculation of the maximum instantaneously achieved value.

Figure 3 shows the corresponding results when the action of the speed brake is present. Apparently, the braking force has no influence at all on $A_{t_{max}}$, whereas a slight effect is felt on $\Delta P_S/\Delta t$. This is because as the thrust is abruptly brought to zero, the speed brake takes some time to become effective. Furthermore, to keep the altitude constant and to compensate for the longitudinal moment, the angle of attack is decreased and the induced drag, which is significant at low speed, is reduced. On a short initial time interval these considerations explain why in this case the tangential agility shows very little changes with respect to the brakeless case. Substantially different results may be envisaged by properly timing the combined actions of thrust and speed brake. The small but observable effects on the power-loss parameter of the action of the brake can be explained after the meaning of $\Delta P_S/\Delta t$ as a global definition of agility over an entire maneuver is taken into consideration.

In the second application of an inverse simulation, a functional law is assigned to $A_t(t)$ in the form $A_{t_{des}} = A_{t_{max_{des}}}[1 - \cos b(t - t_0)]/2$, $t_0 \le t \le t_0 + 2\pi/b$, and the maximum value of A_t effectively realized by the aircraft model is determined by varying the frequency b in the range of obtainable aircraft maneuvers. In other words, this heuristic approach consists of forcing the aircraft, at a constant altitude and in the symmetry plane, to execute limit maneuvers to verify its possibilities in terms of A_t, when A_n is zero. In this case the unknown quantities are the control actions δ_E and δ_T, and the constraints are $k_1 = 0$ and $V = V(t)$ as obtained through Eq. (2) and the assumed A_t law. This inverse problem is nominal and the results depend on h. For h = 0, the main results are shown in Figs. 4 and 5. In particular, Fig. 4 shows how the values of b have varied and how the corresponding changes of $A_{t_{max_{des}}}$ and $A_{t_{max}}$, for an initial Mach number M_e, are equal to 0.6.

For b larger than 10 s^{-1}, the maximum axial agility presents a negligible reduction, and this indicates that the maximum performance for the considered sample maneuver has been identified, i.e., $A_{t_{max}} = -1.9$ m s^{-3}. Note also the much higher modulus of $A_{t_{max}}$ in Fig. 3 at M_T = 0.6. In this respect it is

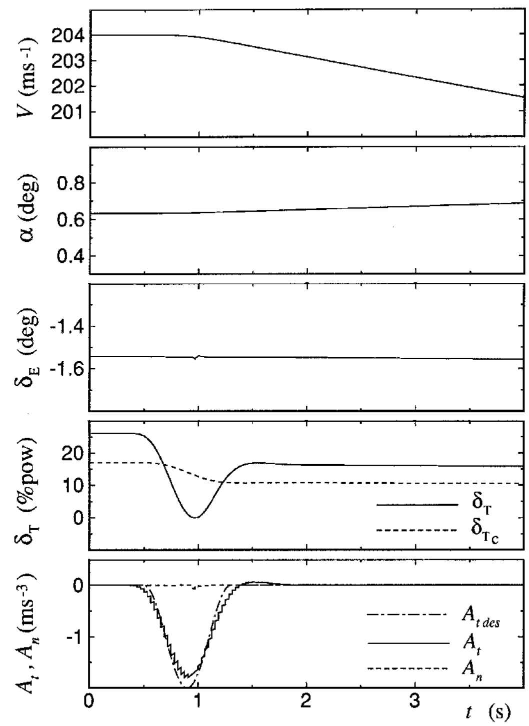

Fig. 5 Time histories for assigned $A_{t_{des}}$; $k_1 = 0$, $V = V(t)$, $b = 7.85$ s^{-1}.

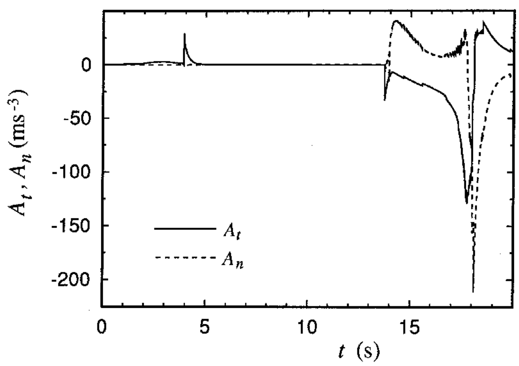

Fig. 6 Agility components vs t; A_n not constrained.

shown that in the former situation the peak performances are maximized and the reported value of $A_{t_{max}}$ is realized after a step-throttle reduction is conducted. In the actual case, we are reasoning in terms of sustained performances because given levels of axial acceleration are to be maintained for a minimum of a few seconds.

For $b = 7.85\ \mathrm{s}^{-1}$, Fig. 5 reports the final optimal solutions as far as the state variables, the relative control actions, and the tangential and normal agility components are concerned. Note that A_n is vanishing. Note also that the maximum values of A_t in this case are much less than those obtained in the preceding situation that are reported in Fig. 3. The problem, however, raises some interest because it shows the capability of determining the control actions when the agility law itself is imposed.

After relaxing the conditions for a rectilinear flight path, the same maneuver of the first case was dealt with as a redundant problem. After attaining M_T the condition of vanishing curvature is no longer imposed and the cost function $S = 10^3/A_t^2 + 10^{-3}A_n^2 + 10e^{-10\sin\gamma}$ is assigned with the objective of maximizing the local value of A_t as realized along the trajectory and not allowing A_n to assume too high a value. The third term in the preceding expression of S is intended to force the aircraft

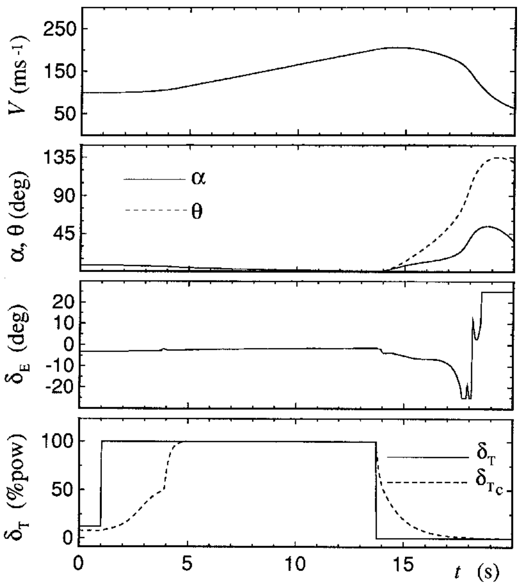

Fig. 7 Time histories for the case with A_n not constrained.

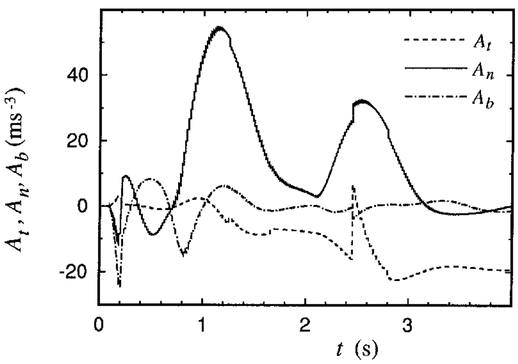

Fig. 8 Agility components vs t in a maximum performance turn; $V = V_c$, $h = 0$, $\dot{\psi}$ maximized.

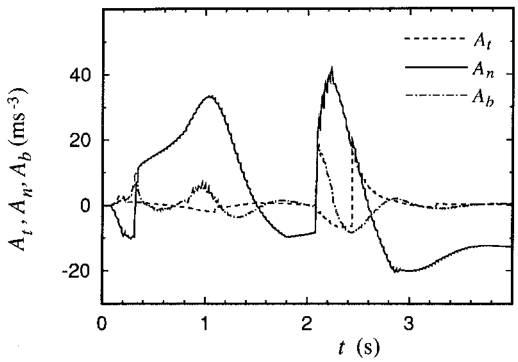

Fig. 10 Agility components vs t in a maximum performance turn; $A_t = 0$, $h = 0$, $\dot{\psi}$ maximized.

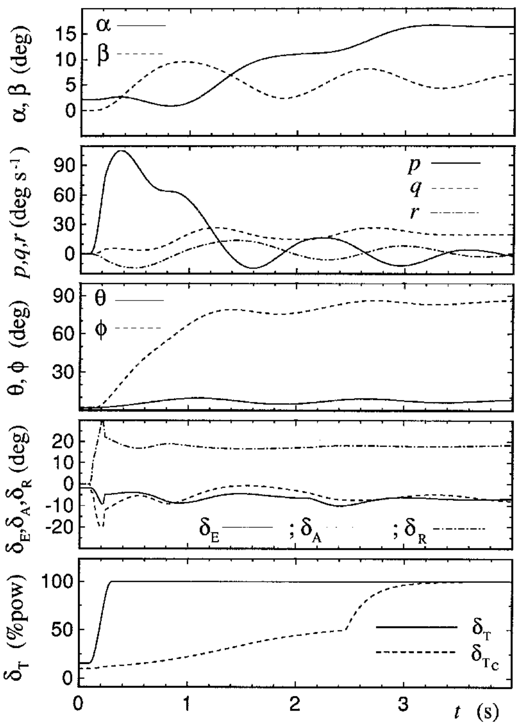

Fig. 9 Time histories for a maximum performance turn; $V = V_c$, $h = 0$, $\dot{\psi}$ maximized.

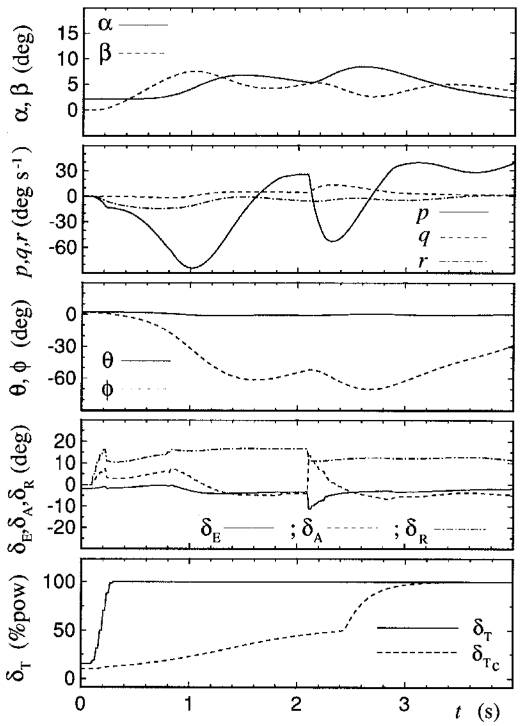

Fig. 11 Time histories for a maximum performance turn; $A_t = 0$, $h = 0$, $\dot{\psi}$ maximized.

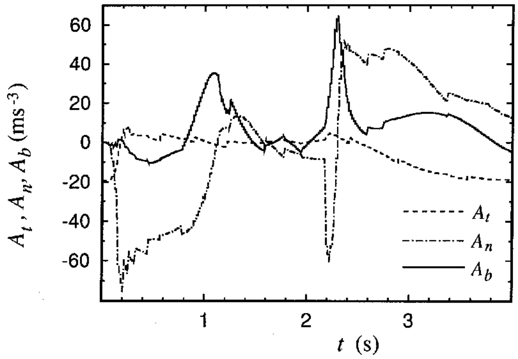

Fig. 12 Agility components vs t in a turn reversal; $\phi = \phi(t)$.

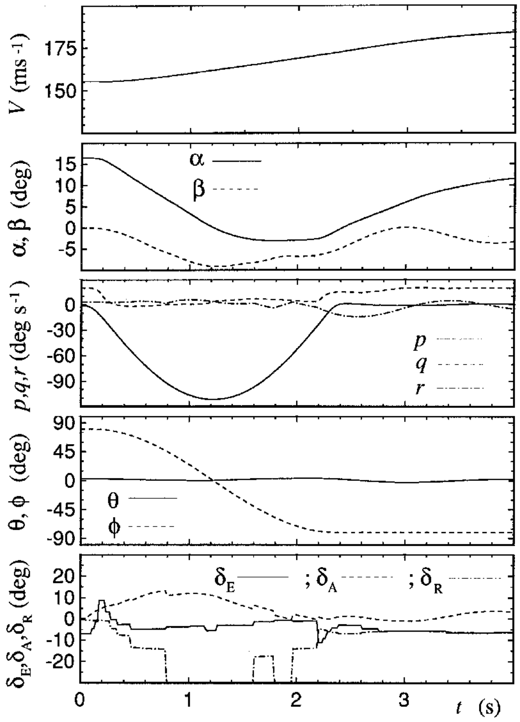

Fig. 13 Time histories for a turn reversal; $\phi = \phi(t)$.

to pitch up during the deceleration. As expected, because k_1 is no longer constrained to zero, the results of Figs. 6 and 7 show that the value of the tangential agility is greatly increased when compared to the data reported in Fig. 1. Note also the high value of A_n at the end of the simulation caused by the abrupt variation of the elevator angle for trimming the aircraft at a high angle of attack.

We proceed to the case of searching for the maximum of A_n according to the commonly accepted maneuver of Ref. 4. This maneuver corresponds to a maximum performance turn at constant $h = 0$ so that $A_b = 0$. We start from trimmed rectilinear flight conditions at the corner velocity $V_c = 155.5$ ms^{-1}, then the thrust is increased to its maximum, the speed and the altitude are kept constant, and A_n and A_t are calculated along the turn. The unknowns are δ_A, δ_E, and δ_R and the inverse problem has one degree of redundancy. Following the procedure of local optimization to solve these kind of problems, a penalty function is assumed to maximize the rate of change of the heading angle $\dot{\psi}$ as the aircraft turns at an increasing load factor. The results concerning the agility components and the time histories of the relevant state and control variables are reported in Figs. 8 and 9, respectively. These results can then be compared with those in Figs. 10 and 11, which were obtained at a speed by imposing that was not constant and where the turn is executed at $A_t = 0$, $A_b = 0$, while still keeping locally maximum $\dot{\psi}$. Here one should observe that the maximum A_n values are greater when the value of A_t is not constrained to be zero. Note in Fig. 10 that the constraint $A_t = 0$ could not be satisfied at $t \simeq 2.1$ s because of the delay in the ignition of the afterburner.

A turn-reversal maneuver was carried out on our model aircraft as a final application for the evaluation of the torsional agility characteristics. The initial state was a sustained turn at the maximum installed thrust with afterburning at the corner speed and an initial angular velocity of $\omega_e = 21$ deg s^{-1}. A law for the roll angle was assigned, $\phi = \phi_e \cos[\pi(t - t_0)/T]$, with $\phi_e = 80.3$ deg and $T = 2.25$ s. The problem is doubly redundant because the unknown control actions were δ_E, δ_A, and δ_R, and it was proposed that $(\boldsymbol{\omega} - \boldsymbol{\omega}_e)^2$ be kept to a minimum along the trajectory. The results are shown in Figs. 12 and 13 and one can observe that the aircraft keeps an almost constant altitude during this maneuver, presenting extremely high values of the torsional and normal agilities.

Conclusions

In this paper we adopted the agility metric based on the definition of $\boldsymbol{A}$ as the derivative of the acceleration vector. This may be considered a choice that depends on the specific purpose for which the evaluation of this parameter is needed. Other proposals were presented to tailor the agility metric definition according to the practical situations where the flight qualities of an aircraft are to be evaluated. However, it is the authors' opinion that the present work follows a rigorous approach that is founded on the differential geometry of the aircraft flight trajectories. The problem of determining the agility characteristics of an airplane is dealt with as an inverse problem either from the point of view of the numerical simulation or for envisaging pertinent flight tests. In most of the reported applications, our principal concern was to consider maneuvers where one component of the agility vector either greatly prevails on the other two or can possibly be determined alone.

A specific conclusion from the applications in this work is that the constraints of keeping two components of $\boldsymbol{A}$ equal to zero lead to the evaluation of smaller maximum values of a third component in the case where these constraints are relaxed. However, this should be interpreted as a way to provide reference conditions for comparing aircraft performances.

A second conclusion is that the method recovers results obtained via different metrics and interprets them in a more general framework. The procedure can be extended to long-time maneuvers where a sequence of specific tasks is performed to appreciate the overall agility characteristics of a high-performance aircraft. As a simple example of one of these tasks the turn-reversal maneuver was discussed.

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